Electromagnetic Wave Propagation Simulation in Corrugated Waveguide with Miter Bend by FDTD: Influence of Induced Current Density

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The optimal corrugated grooves in miter bend are verified by electromagnetic wave propagation simulation using Finite-Difference Time-Domain (FDTD) method, and the effect of induced current density in the waveguide wall is implemented by Drude model. From the simulation results, after bending electromagnetic wave at miter bend, the transfer mode was distorted by reflection at the miter bend mirror. After transmitting in the corrugated waveguide, the transfer mode that is distorted was transitioned to the eigenmode. In addition, the transmission efficiency strongly depends on the corrugated grooves. The parameters of the structure of the corrugated grooves that are determined experimental empirically were high transmission efficiency by the simulation.

Index Terms—**Electromagnetic propagation, Miter Bend, Numerical simulation, FDTD, Drude model.**

I. Introduction

The Electron Cyclotron Heating (ECH) system is used for the plasma heating in the Large Helical Device (LHD) [1]. In ECH, the electrical power that is made by the gyrotron system transmits to LHD by long corrugated waveguide. The corrugated waveguide is bent by miter bend because it cannot be connected directly through the wall separating the gyrotron system and LHD. The miter bend is primary cause of the transmission loss. The optimum shape of the corrugated groove is not sufficiently verified.

In the previous study, the transmission efficiencies of the corrugated waveguide and miter bend have been investigated [2]-[3]. In order to investigate the transmission efficiencies, the perfect electric conductor (PEC) was adopted as the metal. However, the Joule heating was not generated because PEC has no electric resistance. It is known that the Drude model can be substantiated the complex dielectric constant for the metal as a waveguide wall [4].

The purpose of the present study is to develop the numerical code for analyzing the wave propagation phenomena in the corrugated waveguide by FDTD with induced current density on the waveguide wall. In addition, the influence of the miter bend on the mode transfer is also investigated.

II. Implementation of Induced current density

The Finite-Difference Time-Domain (FDTD) method is widely used in the numerical analysis of the electromagnetic field simulation. Maxwell's equation can be solved directly in FDTD. In addition, physical model can be easily achieved by FDTD because of the simple algorithm.

In this study, the Drude model is adopted to represent the metal strictly. In the Drude model, the relative permittivity ε_r is defined by

$$\varepsilon_r(\omega) = 1 - \frac{\omega_{\mathrm{pe}}^2}{\omega(\omega - j\gamma)} \tag{1}$$

where ω_{pe} denotes the plasma frequency and γ denotes the inverse of the relaxation time. The update equation in the metal is calculated using the inverse Fourier transforms of (1) because of the angular frequency function $\varepsilon_r(\omega)$. The inverse Fourier transforms of (1) is discretized by the recursive convolution (RC) algorithm [5]. Finally, FDTD with Drude model is updated using following equations:

$$\begin{aligned}\boldsymbol{E}^n(\boldsymbol{x}) = {} & \frac{\varepsilon_\infty}{\varepsilon_\infty - \chi^0}\boldsymbol{E}^{n-1}(\boldsymbol{x}) + \frac{1}{\varepsilon_\infty - \chi^0}\boldsymbol{\phi}^{n-1}(\boldsymbol{x}) \\ & - \frac{\Delta t}{\varepsilon_0(\varepsilon_\infty - \chi^0)}\boldsymbol{J}^{n-\frac{1}{2}}(\boldsymbol{x}) \\ & + \frac{\Delta t}{\varepsilon_0(\varepsilon_\infty - \chi^0)}\nabla \times \boldsymbol{H}^{n-\frac{1}{2}}(\boldsymbol{x}),\end{aligned} \tag{2}$$

$$\boldsymbol{H}^{n+\frac{1}{2}}(\boldsymbol{x}) = \boldsymbol{H}^{n-\frac{1}{2}}(\boldsymbol{x}) - \frac{\Delta t}{\mu}\nabla \times \boldsymbol{E}^n(\boldsymbol{x}). \tag{3}$$

Here, $\boldsymbol{E}$ denotes the electric field, $\boldsymbol{H}$ denotes the magnetic field and $\boldsymbol{J}$ denotes the induced current density. Moreover, n denotes the step number, ε_∞ denotes the permittivity in case of $\omega = \infty$, ε_0 denotes the permittivity in vacuum and Δt denotes the time step. The convolution summation $\boldsymbol{\phi}^{n-1}(\boldsymbol{x})$ and the initial value of the relative electric susceptibility χ^0 are defined by the following equations:

$$\boldsymbol{\phi}^{n-1}(\boldsymbol{x}) = \Delta\chi^0 \boldsymbol{E}^{n-1}(\boldsymbol{x}) + e^{-\gamma\Delta t}\boldsymbol{\phi}^{n-2}(\boldsymbol{x}), \tag{4}$$

$$\boldsymbol{\phi}^0(\boldsymbol{x}) = 0, \tag{5}$$

$$\chi^0 = \frac{\omega_{\mathrm{pe}}^2}{\gamma}(\Delta t - \frac{1}{\gamma}(1 - e^{-\gamma\Delta t})^2). \tag{6}$$

Furthermore, $\Delta\chi^0$ is defined as

$$\Delta\chi^0 = \frac{\omega_{\mathrm{pe}}^2}{\gamma^2}(1 - e^{-\gamma\Delta t})^2. \tag{7}$$

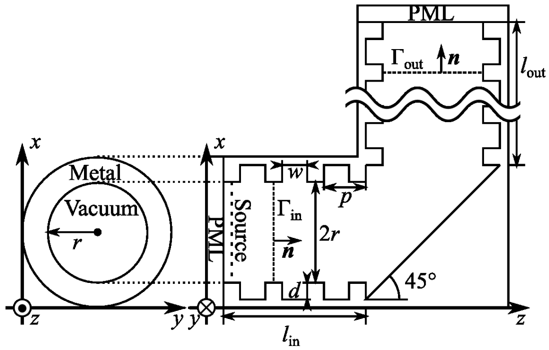

Fig. 1. The analytic model of the miter bend.

The electromagnetic field in the metal is updated by (2), (3) which are adapted Ohm's law. Here, the electric conductivity σ is defined by

$$\sigma = \frac{\varepsilon_0 \omega_{\mathrm{pe}}^2}{\gamma} \tag{8}$$

from the Drude model.

III. Results and Discussion

By the miter bend (see Fig. 1), the electromagnetic wave that propagated corrugated waveguide is bent at a right angle by 45 degrees tilted mirror. A corrugated grooves digs periodically in waveguide wall perpendicular to the propagating direction of wave.

In ECH experiments, HE_{11} mode that is an eigenmode of the corrugated waveguide is adopted as the propagation mode [6]. The effect of the waveguide wall is suppressed in the HE_{11} mode because the electric field is highest concentration at the center of the waveguide. In order to implement experiment fact on the numerical simulation, following assumptions are employed: the input mode is HE_{11} mode, the input frequency is 84 GHz, $l_{\mathrm{in}} = 3.6$ mm, $l_{\mathrm{out}} = 108$ mm, the waveguide consists of the aluminum and inside of the waveguide is vacuum. In addition, assuming the waveguide extends to infinity, Perfect Matched Layer (PML) that is the absorbing boundary condition is placed at both ends of the waveguide.

The distribution of the electric field intensity in the miter bend is shown in Fig. 2 for $p = 1.4$ mm, $w = 1.0$ mm and $d = 0.8$ mm. The electric field intensity of the input plane was affected by reflection from the mirror (see Fig. 2(b)). Similarly, the electric field intensity of the middle cross-section was affected by reflection (see Fig. 2(c)). On the other hand, the input mode that is HE_{11} mode was observed at output plane (see Fig. 2(d)). The distribution of the electric field intensity in the middle cross-section is converted to the eigenmode of corrugated waveguide.

The value of the transmission efficiency R_{TE} is defined by the following equation:

$$R_{\mathrm{TE}} = \frac{\left\langle \int_{\Gamma_{\mathrm{out}}} |\boldsymbol{P}| \, d\Gamma \right\rangle_t}{\left\langle \int_{\Gamma_{\mathrm{in}}} |\boldsymbol{P}| \, d\Gamma \right\rangle_t}. \tag{9}$$

Here, Γ_{in} denotes the input plane, Γ_{out} denotes the output plane, $\boldsymbol{P}$ denotes the Poynting vector. The influence of the corrugated grooves on the transmission efficiency is shown in Fig. 3. We can see from this figure that the electric power is hardly attenuated in case of Fig. 3(a). The parameters of Fig. 3(a) are used in LHD experiment. On the other hand, the attenuation of the electric power can be seen in case of Fig. 3(b) and Fig. 3(c). From the above results, the transmission efficiency depends on shape of the corrugated grooves.

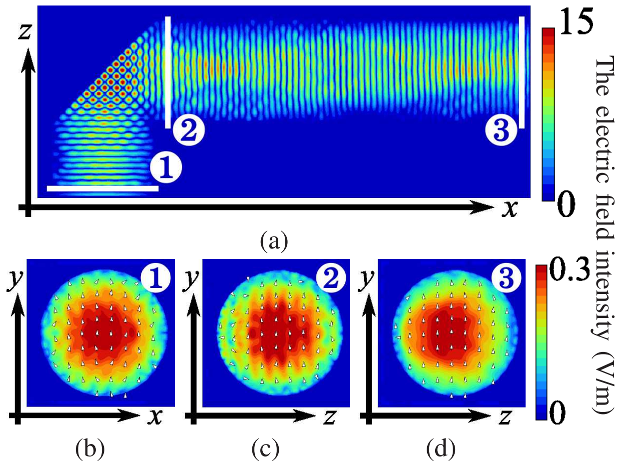

Fig. 2. The distribution of the electric field intensity at the miter bend. Here, (a) is cross-section of the miter bend, (b) is input plane, (c) is middle of the cross-section and (d) is output plane. In addition, (a) is $|\boldsymbol{E}|$, (b), (c) and (d) are $\langle |\boldsymbol{E}| \rangle_t$, respectively.

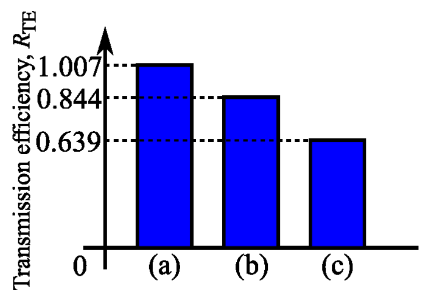

Fig. 3. The influence of the corrugated grooves on the transmission efficiency. Here, (a): $p = 1.4$ mm, $w = 1.0$ mm and $d = 0.8$ mm, (b): $p = 1.4$ mm, $w = 1.0$ mm and $d = 1.8$ mm, (c): $p = 3.6$ mm, $w = 1.0$ mm and $d = 0.8$ mm.

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